Scott Wilkie: Progression in Number Fluency

To have a real impact on number fluency across the school, practise needs to be daily and expectations need to be high. Children need to be able to use their year group strategies by the end of the year and interventions should be used to fill any gaps. Teaching staff may need to look back at previous years to begin with until the strategies are embedded.

Every week should have a different fluency strategy focus. There are roughly 10 and they should be revisited once you have completed them to embed firmly. Children only need a couple of questions of this type a day and it should be extremely quick and mental by definition. Each class should have a fluency grid and a times tables grid and should keep a note of children that have mastered that strategy/table and those who have not.

Key questions for teachers to ask include:

- Which is the most efficient method?
- Which method will get you the correct answer quickest?
- How did you work that out?
- Who did it another way?
- Which is the easier/easiest way?
- Why?
- How is this similar/different to the other way?
- Would this method always work?
- How could you show this with resources/images/bar?

Opportunities to practise fluency:

- Mental maths slot at start of lesson
- Mental maths at start of pm
- During register
- As children line up to

Helpful Links

https://nrich.maths.org/10624 What is fluency? Useful N-rich links

http://www.sharingbestpractice.camden.gov.uk/case-study/number-fluency/ Number fluency case study

https://www.kenkenpuzzle.com/?redirected=1# 4 operations and times table activity

https://www.twinkl.co.uk/resources/ks1-maths-resources-year-1 NC fluency links and activities KS1

https://www.twinkl.co.uk/resources/ks2-maths-2014-year-3 NC fluency links and activities KS2 https://www.theschoolrun.com/what-are-number-facts Teaching number facts ideas

What is fluency?

The first thing to say is that fluency is not only about number – there are other areas of the curriculum where fluency is important. However it's probably sensible to acknowledge that number is by far the largest part of the primary curriculum, so in this article we'll concentrate on that. We're not the only nation to take a recent interest in this – in the US the new standards have quite a lot to say about being fluent:

Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently.

Russell (2000) spells this out in more detail and suggests that fluency consists of three elements:

Efficiency - this implies that children do not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.

Accuracy depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.

Flexibility requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the numbers involved, and also be able to use one method to solve a problem and another method to check the results.

So fluency demands more of students than memorising a single procedure – they need to understand *why* they are doing what they are doing and *know when it is appropriate* to use different methods.

Year 1 Number Fluency Expectations

National Curriculum Expectations	Mental Calculation Strategies
Pupils should be taught to: -count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number -count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens -given a number, identify one more and one less -represent and use number bonds and related subtraction facts within 20 -add and subtract one-digit and two-digit numbers to 20, including zero <u>Times Tables</u> 1,2,3,5,10	<u>a) Counting forwards and backwards in ones or twos</u> 4 + 8 count on in ones from 4 or count on in ones from 8 7 – 3 count back in ones from 7 13 + 4 count on from 13 15 – 3 count back in ones from 15 18 – 6 count back in twos <u>b) Reordering by putting the larger number first</u> 2 + 7 = 7 + 2 5 + 13 = 13 + 5 <u>c) Reordering to find number bonds</u> 3 + 4 + 7 = 3 + 7 + 4 <u>d) Compensating to add 9</u> 5 + 9 = 5 + 10 – 1 <u>e) Using Near Doubles</u> 5 + 6 is double 5 and add 1 or double 6 and subtract 1 <u>f) Bridging through 10 and later 20 when adding a single-digit number</u> 17 + 6 = 17 + 3 + 3 <u>g) Use patterns of similar calculations</u> 7+ 8 is 15 so 27 + 18 will have 5 ones <u>h) Estimating for checking</u> Know that 7 + 9 will be between 10 and 20 Know that doubled numbers will be even, numbers multiplied by 5 will have 5 or 0 in ones column <u>i) Know number bonds for all numbers up to 10</u> 7 = 1 + 6 = 2 + 5 = 3 + 4 = 4 + 3 = 5 + 2 = 6 + 1

Year 2 Number Fluency Expectations

National Curriculum Expectations	Mental Calculation Strategies
-count in steps of 2, 3 and 5 from 0, and in tens from any number, forward and backward -identify, represent and estimate numbers using different representations, including the number line -use place value and number facts to solve problems -recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 -add and subtract numbers mentally including: -a two-digit number and ones -a two-digit number and tens -two two-digit numbers -adding three one-digit numbers -show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot -recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems -estimate to check their answers to a calculation are reasonable (e.g. knowing 48 + 35 is less than 100) Times Tables 1,2,3,4,5,6,8,10	a) Counting forwards and backwards in tens, ones or a suitable multiple 14 + 3 count on in ones from 14 27 – 4 count on or back in ones from any two-digit number 18 – 4 count back in twos from 18 30 + 3 count on in ones from 30 b) Find a small difference by counting up from the smaller to the larger number 32 – 28 count on from 28 c) Reordering to start with larger number 2 + 36 = 36 + 2 d) Reordering to look for number bonds 5 + 7 + 5 = 5 + 5 + 7 e) Partitioning using multiples of 10 and 100 30 + 47 = 30 + 40 + 7 78 – 40 = 70 – 40 + 8 25 + 14 = 20 + 5 + 10 + 4 = 20 + 10 + 5 + 4 f) Partitioning – bridging through multiples of 10 6 + 7 = 6 + 4 + 3 23 – 9 = 23 – 3 – 6 15 + 7 = 15 + 5 + 2 g) Compensating to add or subtract 9 or 19 34 + 9 = 34 + 10 – 1 52 + 19 = 52 + 20 - 1 70 – 9 = 70 – 10 + 1 h) Partitioning into '5 and a bit' to add 6, 7 or 8 15 + 7 = 15 + 5 + 2 i) Use the relationship between addition and subtraction I know 8 + 7 = 15 so I know 15 – 8 = 7 j) Multiplying by 10 moves one place value column to the left 3 ones x 10 = 3 tens = 30

	2 tens x 10 = 2 hundreds = 200

National Curriculum Expectations	Mental Calculation Strategies

-count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
-recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
-compare and order numbers up to 1000
-identify, represent and estimate numbers using different representations
-add and subtract numbers mentally, including:
    -a three-digit number and ones
    -a three-digit number and tens
    -a three-digit number and hundreds
-recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
-write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written method
-estimate the answer to a calculation and use inverse operations to check answer
 	
Times Tables
All to 12 x 12
	a) To count on in different jumps bridging 10s and 100s
	Count in 10s from 76
	Count in 2s from 15
	b) Reordering and finding number bonds or near doubles to add multiple numbers
	23 + 54 = 54 + 23
	12 – 7 – 2 = 12 – 2 – 7
	13 + 21 + 13 = 13 + 13 + 21 (using double 13)
	c) Partitioning using multiples of 1, 10 and 100
	23 + 45 = 40 + 5 + 20 + 3= 40 + 20 + 5 + 3
	68 – 32 = 60 + 8 – 30 – 2= 60 – 30 + 8 – 2
	d) Partitioning – bridging through multiples of 10
	49 +32 = 49 +1 +31
	e) Compensating to add or subtract 8,9, 18, 19 etc.
	53 + 9 = 53 + 10 - 1
	84 – 18 = 84 – 20 + 2
	f) Using Near Doubles
	18 + 16 is double 18 and subtract 2 or double 16 and add 2
	36 + 35 is double 36 and subtract 1 or double 35 and add 1
	60 + 70 is double 60 and add 10 or double 70 and subtract 10
	g) Using a known fact to identify others
	7 + 8 = 15 so 15 – 7 = 8
	7 + 8 = 15 so 7 + 28 = 15 + 20 =35
	h) Multiplying by 10/100 moves one/two place value column to the left
	3 ones x 10 = 3 tens = 30
	2 tens x 100 = 2 thousands = 2,000
	i) To use estimations/number facts to check accuracy
	46 + 58 will be close to 100
	23 x 5 will end in 5 or 0
	j) to know the relationship between multiplication and division fact families
	3 x 6 = 18 so 18 ÷ 3 = 6 and 18 ÷ 6 = 3

Year 4 Number Fluency Expectations

National Curriculum Expectations

-count in multiples of 6, 7, 9, 25 and 1000
-find 1000 more or less than a given number
-count backwards through zero to include negative numbers
-round any number to the nearest 10, 100 or 1000
-solve number and practical problems that involve all of the above and with increasingly large positive numbers
-estimate and use inverse operations to check answers to a calculatio
call multiplication and division facts for multiplication tables up to 12 × 12
-use place value, known and derived facts to multiply and divide mentally, including:
- -multiplying by 0 and 1
- -dividing by 1
- -multiplying together three numbers

-recognise and use factor pairs and commutativity in mental calculations
-count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
-find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredth

Times Tables

All to 12x12

Mental Calculation Strategies

a) To count on and back in different jumps bridging 10s, 100s and 100's
Count in 10s from 76
Count in 2s from 15
86 – 30 count back in tens from 86 or count on in tens from 30
960 – 500 count back in hundreds from 960 or count on in hundreds from 500

b) Reordering to add or multiply 3 or more numbers
6 + 13 + 4 + 3 = 6 + 4 + 13 + 3
17 + 9 – 7 = 17 – 7 + 9
4 x 3 x 5 = 5 x 4 x 3 = 20 x 3 = 60

c) Partitioning – bridging through multiples of 10 or 100
57 + 14 = 57 + 3 + 11 or 57 + 13 + 1

d) Compensating to add or subtract
38 + 69 = 38 + 70 – 1
53 + 29 = 53 + 30 – 1
64 – 19 = 64 – 20 + 1

e) Using Near Doubles
38 + 35 is double 35 and add 3
160 + 170 is double 150 and add 10 then add 20, or double 160 and add 10, or double 170 and subtract
380 + 380 is double 400 and subtract 20 twice

f) Identify fact families for addition and subtraction to solve problems
I know 6 x 7 = 42 so I know 42 ÷ 6 = 7

h) Multiplying by 10/100 moves one/two place value column to the left and dividing moves to the right
3 ones x 10 = 3 tens = 30
2 tens x 100 = 2 thousands = 2,000

i) Using partitioning to multiply
17 x 6 = 10 x 6 + 7 x 6 = 60 + 42 = 102

j) Partioning to double or halve any number
Double 347 is double 300, double 40 and double 7
Half of 450 is half 400, half 40, half 10

Year 5 Fluency Expectations

National Curriculum Expectations	Mental Calculation Strategies
-read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit -count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 -interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero -round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 -add and subtract numbers mentally with increasingly large numbers -use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy -identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers -know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers -multiply and divide numbers mentally drawing upon known facts -multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3) Times Tables All to 12 x 12 Identify square numbers to 12 x 12 Identify prime numbers up to 20	a) Counting backwards and forwards in multiples of 1 or 10 from any number to 1 000 000 Count in 20s from 346 b) Partitioning to add – bridging through multiples of 1, 10, 100 or 1,000 3.8 + 2.6 = 3.8 + 0.2 + 2.4 560 + 357 = 560 + 40 + 317 c) Compensating to add or subtract any near multiples of 10 or 100 138 + 69 = 138 + 70 – 1 405 – 399 = 405 – 400 + 1 21/2 + 13/4 = 21/2 + 2 – [illegible] d) To be able to count backwards across 0 in different jumps Count back from 0 in 2s e) Finding all factors of a number and identify fact families I know 24 is divisible by 1, 2, 3, 4, 6, 8, 12, and 24 f) Multiplying by 10/100 moves one/two place value column to the left and dividing moves to the right 3 ones x 10 = 3 tens = 30 2 tens x 100 = 2 thousands = 2,000 gi) Using partitioning to multiply 17 x 6 = 10 x 6 + 7 x 6 = 60 + 42 = 102 h)To partition to multiply by multiples of 10, 100 etc 43 x 30 = 43 x 3 x 10 or 43 x 10 x 3 25 x 400 = 25 x 4 x 100 i)To be able to identify the nearest multiple of 10, 100, 1,000 578 is closest to 600 8243 is closest to 8,000 j) Estimate answers by rounding and using number facts 687 + 503 will be roughly 700 + 500 =1,200 67 x 38 will be roughly 70 x 40 =280 876 ÷ 5 will have a remainder

Year 6 Number Fluency Expectations

National Curriculum Expectations	Mental Calculation Strategies
Consolidate all above -read, write, order and compare numbers up to 10 000 000 and determine the value of each digit -round any whole number to a required degree of accuracy -use negative numbers in context, and calculate intervals across zero -perform mental calculations, including with mixed operations and large numbers -identify common factors, common multiples and prime numbers -use their knowledge of the order of operations to carry out calculations involving the four operations -use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy Times Tables All to 12 x 12 Identify square numbers to 12 x 12 Identify cube numbers up to 5 x 5 x 5 Identify prime numbers up to 100	a) Counting backwards and forwards in multiples of 1 or 10 from any number to 1 000 000 Count in 20s from 346 b) Partitioning to add – bridging through multiples of 1, 10, 100 or 1,000 3.8 + 2.6 = 3.8 + 0.2 + 2.4 560 + 357 = 560 + 40 + 317 c) Compensating to add or subtract any near multiples of 10 or 100 138 + 69 = 138 + 70 – 1 405 – 399 = 405 – 400 + 1 21/2 + 13/4 = 21/2 + 2 – □ d) To be able to count backwards across 0 in different jumps Count back from 0 in 2s e) Multiplying by 10/100 moves one/two place value column to the left and dividing moves to the right etc. 3 ones x 10 = 3 tens = 30 2 tens x 100 = 2 thousands = 2,000 fi) Using partitioning to multiply 17 x 6 = 10 x 6 + 7 x 6 = 60 + 42 = 102 g)To partition to multiply by multiples of 10, 100 etc 43 x 30 = 43 x 3 x 10 or 43 x 10 x 3 25 x 400 = 25 x 4 x 100 h)To be able to round to the nearest multiple of 10, 100, 1,000, etc. 578,745 is closest to 600,000 8243 is closest to 8,000 i) To be able to count in tenths, hundredths and thousandths and know when to rename 0.7, 0.8, 0.9, 1 0.695, 0.696, 0.697, 0.698, 0.699, 0.7 j) Estimate answers by rounding and using number facts 687 + 503 will be roughly 700 + 500 =1,200 67 x 38 will be roughly 70 x 40 =280 876 ÷ 5 will have a remainder

Why do children need to be fluent?

To the person without number sense, arithmetic is a bewildering territory in which any deviation from the known path may rapidly lead to being totally lost. Dowker (1992)

The phrase 'number sense' is often used to mean conceptual fluency – understanding place value and the relationships between operations. Children need to be both procedurally and conceptually fluent – they need to know both how and why. Children who engage in a lot of practice without understanding what they are doing often forget, or remember incorrectly, those procedures. Further, there is growing evidence that once students have memorised and practised procedures without understanding, they have difficulty learning later to bring meaning to their work (Hiebert, 1999).

Russell describes two instances where children had a good idea about number relationships and operations but failed to use these successfully in practice. I'm sure you can think of similar examples that you have seen.

Child A knew, *when asked verbally*, what 112 and 40 were, and she had strategies to work out the answer which indicated that she understood place value – add 40 onto 110 and then add on the extra 2. But when asked to do it as a written calculation, she remembered an algorithm which was to do with lining up the numbers - and she remembered it incorrectly.

Child A		Child B	
	112		$\overset{2}{5}7$
	+40		x 4
	512		288

Similarly Child B could work out 57 x 4 *mentally* using the knowledge that 57 is 50 and 7 and breaking down the calculation into 50 x 4 and adding on 7 x 4. But he had remembered a written algorithm which was to do with carrying a digit over - and he remembered it incorrectly. (Can you see what he did? He added the 2 to the 5 before multiplying it by 4.) Both children knew their written answers were not correct but were convinced they had used the right method (and you might wonder what instructions they rehearsed in their heads which led them to believe that).

On the other hand, conceptual fluency without procedural fluency can make the problem-solving process tortuous – children lose track of their thinking because they have to divert their energies into calculations which should be quick but aren't.

How can we support children in becoming fluent?

As with much of mathematics, the key to fluency is in making connections, and making them at the right time in a child's learning.

Manipulatives

We learn by moving from the concrete to the abstract and structured apparatus such as Dienes can be helpful for learning about place value or number bonds. However the meaning isn't in the manipulatives themselves – it has to be constructed by children over a period of time, through playing around with them and connecting them directly to mental and recorded calculation.

Talking about their work

At NRICH we often say you can't do maths unless you talk maths. But the quality of the talk is important. It is not simply children sharing how they did a particular calculation, but describing why and how it worked, and how their method is the same or different to those of others. In other words, giving children opportunities to use those higher-level skills of comparing, explaining and justifying. Russell says 'The reason that one problem can be solved in multiple ways is that mathematics does not consist of isolated rules, but connected ideas. Being able to solve a problem in more than one way, therefore, reveals the ability and the predilection to make connections between and among mathematical areas and topics'.

Consolidation in meaningful contexts

By offering children practice in context we help them to make links between the types of situations that a particular strategy might suit. Russell calls this mathematical memory, which is different from just memorising. She says that important mathematical procedures cannot be "forgotten over the summer" because they are based in a web of connected ideas about fundamental mathematical relationships.

Child	a	b	c	d	e	f	g	h	i	j